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On the Improvement of Life Contingency Calculation. Part II.: The System of Dependent Risks. *By* Edwin James Farren, *Fellow of the Statistical Society and of the Institute of Actuaries.*

[Read before the Institute of Actuaries, 29th November, 1858, and ordered by the Council to be printed.]

IN the former paper,* it was laid down, as the basis of any improvements I might be able to suggest as to life contingency calculation, that, in place of premises of a specific nature leading to special conclusions, we should rather, from the very commencement, recognise the variable nature of the results we might have to encounter, and accordingly construct a system of limits rather than one of particular values.

Speaking geometrically, the prevailing doctrine starts with the assumption that the so-called law of mortality, and, indeed, of interest of money, can be represented by a single line, and that its exact course can be predicated. The effect of such an assumption is at once to present the system as one wholly confined to one particular branch of the exact sciences; the resulting values being, in fact, no more than the representative details of a certain function dependent on the exact nature of the particular line chosen, and taken as the emblem of the law of mortality itself. Thus we have Halley's Breslau Table virtually symbolized by the harmonical line or theory of De Moivre; Simpson's London Table, by the function of Lambert; the Carlisle Table, by that of Gompertz; the Equitable Experience, by that of Dr. Young; and, incidentally, the Northampton—and, indeed, nearly every other mortality table of fixed values — by various functional or equational contrivances which can practically be substituted for the table itself.

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* *Journal*, vol. v., p. 185.

It is desirable to draw attention very prominently to these substitutions of processes for each other, leading either identically, or in the spirit of identity, to certain fixed results, because it is this very fixity that has falsely given what I have ventured to call elsewhere so "wooden" an appearance to many of our existing processes and calculations.

Speaking also geometrically, the system now proposed, instead of a single line, would require two, between which, sometimes towards the one and sometimes towards the other, the line of results, as they actually occurred from time to time, might be expected to unfold itself, but without any necessary intrinsic property of being exactly predicated. Instead, therefore, of expressing a fixed law of mortality or of interest of money, supposed to be equally prevalent in all Societies and on all occasions, for all periods, however long or however short, the primary conditions, as now proposed, would be fulfilled, whether the law were variable or invariable, or whether there were or were not any primary law whatever, so long as the results themselves, whether in one Society or another, did not fall beyond the expected limits of fulfilment.

It would, therefore, be to these limits, rather than to any primary special values, that our original predications and attention would have to be more particularly directed.

In place of such defined limits, the only substitute we have hitherto had is the application of the term "average" to some favourite special rate, whether of mortality or interest. This favouritism, in some cases, has even been pushed so far as to have induced, as mentioned in the former paper, the adoption and application of the epithets "true rates" or "real rates"* to what should really be considered as merely partial exhibitions of values; essentially variable in their nature, even in the same Society, and by no means, therefore, of such authoritative character as to warrant the assumption of absolutism, or to exclude the adoption of other rates equally "true" or equally "real" in a similarly limited sense.

It is also to be remembered, that the extension now proposed of the premises from which the particular algorithm of life contingencies should be made to flow, is only a change of a cognate kind to that to which the great algorithm of symbolical algebra itself has been made subject even in our own time. Formerly, in following out the algebraical deductions resulting from the ordinary "rule of signs," values were arrived at, which, for the time being, even in the best mathematical treatises, home or foreign, were coupled with no better interpretative terms than those of "imaginary" or "impossible." The remedy for this class of anomalies —as I think it will be found with the anomalies in our own subjects—has been to openly give a greater scope to the original premises themselves, and thereby to include results within the laws of intelligible interpretation that otherwise might be said to be wholly beyond it.

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* *See* "Postscript," vol. v., p. 194.

Not to mention minor anomalies with which the actuary has at present to deal, I shall confine myself to drawing attention to what may be considered the greatest of all, arising out of the prevailing system of calculation, and which has, indeed, prompted the preparation, and forms the object of illustration, of the present paper. I allude to the circumstance of the prevailing system, even as displayed in our text-books, being essentially one merely of prices or premiums, and but little, if at all, one of practical insurance finance.

Tables of prices we have in abundance, even to the remotest decimal; and formulæ for their determination in conjunction with the most unusual combinations of lives and ages; but anything approaching a system for the due appreciation and treatment of results, as they occur from time to time in the same Society or different Societies, it may be fairly said we have not, nor indeed can we have, until our present system of calculation is considerably extended in its first principles, or enlarged by the addition of supplementary methods for the after-finance of phases of results.

It is, indeed, true that a system of calculation which should, in the widest sense of the term "proper," be "proper" to determine the rates of premium or prices to be charged for various risks to be incurred or surrendered, should also be sufficiently comprehensive to include within itself the necessary elements for directing us how to cope financially with the results or risks themselves, after they have been incurred and are in process of occurrence; and did the prevailing system, in its present form, afford this, but little, if any, change of procedure would be needed. But it is the object of the present paper to show that the prevailing system, as commonly explained, is merely one of what may be called "*dependent risks*," from its illustrating in its calculative form but one line of special or predicated results, and not, as a system of *"independent risks"* would do, a variety of results, such as different Societies charging the same premiums, or even the same Society at different periods, has been found to afford.

To treat such a topic as that of dependent and independent risks, in its utmost generality, would be tantamount to writing a treatise on insurance calculation; for it is obvious that fulfilment in the payment of occurred risks, whether originally dependent or independent of each other, is the main object of all Societies, whether for life, fire, marine, or other applications of the doctrine of insurance. Without professing to now attempt such a task, still it is because I hope it may be considered that the time has arrived for a comprehensive treatise to be written, if not by one, at least by a combination of persons; and that life insurance should be definitively shown to be but part of the great and general doctrine of independent risks, that I am anxious, as far as lies within my own power and opportunity, to direct attention, by occasional papers like the present, to the fact of how needlessly life contingency calculation has been restricted in value to insurance managers, by a speciality of premises which only provides formulæ for the determination of prices, and leaves the great questions of financial status almost untouched.

In illustrating the system of dependent risks, as now generally employed, I shall do so by the adoption of a mechanical procedure, thereby showing its confined nature, and therein, as I conceive, sufficiently illustrating its unfitness to grapple with those forms not so much bearing upon the attributes of the exact or abstract sciences as on the sciences of observation, which alone can fairly comprise all the features of the case as presented by the results of the various Societies which now are, or may be hereafter, in practical operation.

The usual illustration for a mechanical system of chances, is that of an urn, as a receptacle from which a variety of balls can be taken. I shall adopt the same illustration in principle; but, to render cases of high numbers somewhat more readily imagined, shall consider the receptacle a common large-sized box for wafers, and the balls the very smallest dark and light-coloured beads, a collection of which, I believe, containing many hundreds, does not exceed a single ounce in weight. We may suppose 10,000 of these to be mixed and placed in a box, and to comprise 8,461 light-coloured and 1,539 dark-coloured beads. The chance of a person blindfolded drawing a light colour would obviously be ·8461, and a dark colour ·1539. If the venture be taken in each case to be for £1, to be payable one year after the drawing, and money to bear 3 per cent., the *immediate* price for the one chance would be ·8461 × ·9708, and for the other ·1539 × ·9708.

If, at the end of the original year, we suppose a new drawing to take place, but to be limited to those who did not draw a dark-coloured bead in the first venture, and the total number of beads to be now limited to 8,461, the former number of the light-coloured, but of which 7,779 only should now be light, and 682 changed for dark-coloured, and the £1 in the second case not to be payable until the end of the second year, then the *immediate* price or premium for the chance of drawing a light colour at the second venture would be ·7779 × ·9425, and of a dark colour ·0684 × ·9425; and pursuing a similar system for a third year, under similar restrictions, using 7,779 as the new total number of beads, of which 7,274 should be light, 505 dark, and discounting for three years, the *immediate* prices or premiums would be ·7274 × ·9151 and ·0505 × ·9151.

It is sufficient to show the principle, to restrict the illustration to the three preceding ventures; but it is obvious that a complete series of successive ventures might also be arranged for each following year, until the original number of 10,000 should be completely exhausted or placed aside, which, in the present instance, may be supposed achieved in 104 drawings or sets after the original.

It is also obvious, that prices or premiums could be arranged for parties wishing to enter at the beginning or midway in the series of ventures, and for the whole series or for a part; that such premiums might be arranged to be made yearly, half-yearly, or quarterly; that premiums for joint ventures, and for contingent ventures of one party drawing a light bead before another should draw a dark one, or *vice versâ*, could also be arranged. That, finally, we might, by mechanical means and under other names, display by aid of any such series of fixed numbers all the usual calculations appertaining to so-called tables of mortality, as now employed and used by Offices.

Further, if we pass from prices or premiums to fulfilments, it is manifest that the grantor of the ventures at fixed ratios can be in need of no capital, and can therefore be under no anxiety as to the discharge of his liabilities, so long as the system be throughout one of "*dependent risks*," that is, a system in which the happening of a defined number of particular risks restricts the possibility of others happening. And it is upon this supposition of speciality of results that tables of mortality, of which the above mechanical illustration is an imitation (the numbers chosen being identical with those of the Carlisle Table), have hitherto been founded, and serves as the virtual plea for the non-inclusion in our text-books on insurance of the consideration of extraneous capital, number of lives concerned, insufficiency or surplusage of funds, and other topics of insurance finance of the highest practical importance.

To show such non-inclusion can only be justified by considering the prevailing system as one of dependent risks, it will only be necessary to glance at the difference of position in which the grantor of the same ventures is immediately placed, if he be openly considered as pledging himself to a system of independent in lieu of that of dependent risks. For example: in the venture of the original 10,000 risks already alluded to, if such be considered as dependent risks, or so arranged that no other result but that of 1,539 dark beads, for instance, can possibly ensue, as would necessarily result if each bead of the 10,000, on being drawn, were put aside before the next were drawn, then not only would the prices quoted be the fair premiums for the risk, but the amount of prices or premiums received would exactly balance the amount demanded in payment of claims. But if the 10,000 risks be rendered independent of each other, as would be the result if each bead, on being drawn, in place of being put aside, were returned to the box prior to the next drawing, so as to keep the risk the same during each set of drawings; then, although the prices or premiums would be identically the same, as a matter of theory, as before, yet, in practice, the results might be essentially different.

For it is apparent, that each person might then draw a light colour, and thus relieve the grantor of the ventures from all payments whatever, if the ventures were on the dark beads; or in the event of each drawing producing a dark colour, a considerable loss would incur beyond the amount of the prices or premiums paid for the ventures; and thus the question of extraneous capital would be at least raised as a necessary topic for consideration in any system of calculation professing to deal with the subject in its entirety.

Without, then, going further at present into the distinctive features of the two systems of dependent and independent risks, it will have been sufficient for the purposes of the present paper to have drawn attention to the material fact, that the prevailing system is really one merely of dependent risks, by its wholly ignoring the necessity of extraneous capital, or even considering the question, and that it therefore does not fairly meet the requirements of Insurance Company problems, fully considered. Further, that to improve such a system, not merely methods of determining the relative prices or premiums for one contingency as compared with another must be involved, but also methods for marshalling results, whether in excess or defect, that may not be in exact accordance with the prices or premiums originally charged.

In a subsequent paper on independent risks, conjoined with a doctrine of limits, I shall at least be able, I trust, to take the first step towards showing that a system, sufficiently combining these requirements, is not, even in practice, wholly beyond our reach. In the mean time, I may be allowed to remark, that, however well the prevailing system may have on occasions appeared to have fulfilled its purpose, it has almost always only done so by including an element upon which dependence can no longer be placed. I allude to what have been called "errors on the safe side," or exaggerations of the primitive ratios. So long as such errors were allowed to remain on the so-called safe side—if errors can ever be said to be safe in any sense—so long a somewhat crude system of calculation might be allowed with some impunity. But when, in the face of competition for public favour, and under the name of the bonus system and distribution of profits, such exaggerations themselves have had, and still have, to be dealt with and distributed, any presumptions of safety that might otherwise have arisen under the plea of excess of caution are in constant danger of becoming elements of evil, by some counter process or procedure which a more comprehensive system of calculation would have rendered unnecessary, or, if necessary, would have legalized.

On the Determination of the Rates of Premium for Assuring against Issue. By ARCHIBALD DAY, *Actuary of the London and Provincial Law Assurance Society and Fellow of the Institute of Actuaries.*

[Read before the Institute, 31st January, 1859, and ordered by the Council to be printed.]

AMONG the numerous classes of risks now undertaken by Insurance Companies, those against the contingency of leaving issue are beginning, from their increasing numbers, to assume a position of greater importance than heretofore. Nothing has hitherto, to my